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SOLID PROPELLANT BURNING RATE MEASUREMENT IN A CLOSED BOMB

Alvars Celmiņš

October 1975

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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Experiments show that the burning rate of solid propellants is a function of the pressure under which the burning takes place. An often used method to determine that burning rate function is to burn a sample of the propellant in a closed container and to register the corresponding pressure-time data. With the aid of formulas from thermodynamic theory these observations can be manipulated to yield the desired parameters of the burning rate function. This analysis is typically done under a number of restrictive assumptions, e.g., the

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assumption of constant temperature in the container throughout the experiment. In spite of such assumptions the corresponding algorithms published are complicated and an error analysis of the results has apparently never been reported. In the present paper new formulas are derived under relaxed assumptions. With the aid of these formulas the data analysis is reduced to a curve fitting task. For such tasks standard theories and computer programs are available, which furnish the unknown parameters of the burning rate function together with estimates of their variances and co-variances. The new formulas can be replaced by simpler ones under conditions, which are controllable by the design of the experiment. It is found that for such simplifications the temperature's behavior is not relevant. In general the simplifications are permissible if the maximum pressure achieved during the experiment remains essentially smaller than the chemical energy released per unit volume of the propellant. The latter is typically of the order of 10^{10} Pa (10^5 atm).

An example illustrates the application of the new algorithm.

to 10th power

10 to 5th power

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LIST OF SYMBOLS

a, A	
b, B	constants, defined by Equations (25) and (34)
c	
C(p)	defined by Equation (41) = dp/dz [Pa]
C^*	constant approximation to C(p) [Pa]
d	constant, defined by Equation (25)
e_o	square root of variance of weight one
e_p	standard error of pressure observations [Pa]
e_t	standard error of time observations [s]
E	internal energy [J]
$\hat{E}$	chemical energy constant, Equation (3) and (37) [Pa]
f	"force constant", Equation (37) [m]
F(V)	defined by Equation (9)
$\hat{f}(m,V)$	factor in the equation of state (1) $[m^{-3}]$
f(V)	factor in the equation of state (6) $[m^{-3}]$
g	standard acceleration = 9.80665 $[m/s^2]$
G	burning rate function, Equation 14 [m/s]
h(z)	factor in the equation of state (27)
m	gas mass [kg]
m_p	initial mass of propellant [kg]
M	molar mass [kg/mol]
n	exponent in burning rate Equation (44)
p	pressure [Pa]
$\hat{p}$	parameter in the burning rate Equation (62) [Pa]

p_{end}	pressure at completed combustion [Pa]
r	linear burning rate [m/s]
R	universal gas constant = 8.31434 [$JK^{-1}mol^{-1}$]
S	surface of the propellant [m^2]
s	integration variable
t	time [s]
T	temperature [K]
V	volume [m^3]
V_{bomb}	volume of combustion chamber [m^3]
V_p	initial volume of propellant [m^3]
v	integration variable [m^3]
W	weighted sum of squares of the corrections
x	linear distance burnt (normal to the surface of the propellant) [m]
$\bar{y}$	parameter vector of burning rate function, Equation (14)
z	fraction of propellant burnt
α	thermal diffusivity, Equation (32) [m^2/s]
β	coefficient in burning rate Equation (44) [$m\ s^{-1}Pa^{-n}$]
γ	ratio of specific heats
η	co-volume [m^3/kg]
ρ	density [kg/m^3]
ρ_p	density of propellant [kg/m^3]
τ	integration variable [s]
Index 0	initial values or state variables describing inert gas
Index 1	state variables describing combustion gas

1. INTRODUCTION

The burning of a solid propellant is known to be a function of the pressure at which the combustion takes place, and also a function of the time derivative of the pressure. One method to determine that function is to burn a sample of the propellant in a closed container ("bomb") and register the pressure-time curve generated in the container by the combustion. Under simple assumptions about the burning process and combustion gases the pressure-time data can be manipulated to yield the desired burning law. Formulas and algorithms which accomplish this task have been published, for instance, in References 1 through 4.

Typically these algorithms are obtained under the assumption of constant temperature throughout the experiment and simple equations of state. (The algorithms may also contain empirical quantities, as in Equation (3) of Reference 2.) Because of their complexity, the published algorithms are difficult to analyze with respect to the confidence limits of the results. As a consequence all burning rate data are published without corresponding error estimates.

In the present paper we shall derive new formulas for the treatment of the pressure-time data, based on somewhat relaxed assumptions. Under conditions which are often satisfied and which can be controlled by the experiment, these formulas transform the problem into a simple curve-fitting task, to which standard error analysis can be applied.

In Section 2 of this paper we shall list and discuss the assumptions on which the analysis of the data is based. A general outline of the method is given in Section 3.

The ideas presented in Section 3 are applied in Section 4 to gases with Noble-Abel equations of state and constant specific heats. That equation of state is frequently used for interior ballistics calculations and often the only equation available for a given propellant. The theory of Section 3 yields for Noble-Abel gases fairly simple formulas.

A further simplification is obtained if the ratio of specific heats of the combustion gases is equal to that of the inert gas which fills

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[1] Wallace, W., "New Formulas for Rapid Calculation of Linear Burning Rates of Solid Propellants", Picatinny Arsenal Technical Report 2488, April 1958.

[2] Fayon, A.M. and Goldstein, F.B., "Evaluation of Solid Propellant Ballistic Properties by Constant Volume Burning", Combustion and Flame <u>10</u>, 23 (1966).

[3] Osborn, J.R., "Evaluation of Solid Propellant Ballistic Properties", Combustion and Flame <u>20</u>, 193 (1973).

[4] Krier, H., Shimpi, S.A., Adams, M.J., "Interior Ballistics Predictions Using Data from Closed and Variable-Volume Simulators", University of Illinois Technical Report AAE 73-6, September 1973.

the bomb at the beginning of the experiment. Formulas for that case are given in Section 5. These formulas show how the pressure rise in the bomb is influenced by the burning rate and by the experimental set-up, respectively. By controlling the latter, the computation of the burning rate can be greatly simplified.

An example for the application of the new formulas to closed bomb measurements is given in Section 6.

2. ASSUMPTIONS

The details of the combustion process and of the associated gas flow in a closed bomb experiment are extremely complicated. For the present purpose we shall, however, neglect particulars of that flow and assume that space average values of state variables describe the state of the gases in the bomb at any time sufficiently accurate. We shall also neglect the kinetic energy versus the internal energy of the gases. These assumptions may be summarized by the following statement (Assumption A):

(A) Combustion gases mix with gases present in the bomb in a time which is short compared with the combustion time.

Our analysis will not cover any chemical reactions and we want to describe the combustion gas by a single equation of state. We make, therefore, the following two assumptions:

(B) The composition of the combustion gas is independent of the conditions in the bomb.

(C) Chemical gas phase reactions take place only within a region arbitrarily close to the surface of unburnt propellant and the gas released fromthat region has a constant (flame) temperature.

We intend to consider a single homogeneous propellant only for which we assume:

(D) The density and the temperature of the unburnt propellant are constant.

The assumption (D) eliminates from our considerations possible effects of a preheating of the propellant during the test. We can summarize the assumptions (B), (C) and (D) as follows:

(E) The energy released by the combustion is proportional to the volume consumed.

We assume also:

(F) Heat losses to the walls of the bomb are negligible.

This assumption is probably the most restrictive one in our analysis. Its purpose to simplify the theory. For similar reasons we require a specific form of the equation of state, namely:

(G) The mixture of gases in the bomb obeys an equation of state of the form,

$$p = E \cdot \hat{f}(m,V) \tag{1}$$

where

- p is pressure
- E is the internal energy of the gases
- m is the gaseous mass
- V is the volume available for the gases, i.e., the inital void volume plus the volume of propellant burned.

The next assumption is a standard assumption about the burning process. Let

- r be the "linear burning rate", which we seek to determine,
- x be the linear distance burnt,
- $S(x)$ be the burning surface of the propellant, and
- t be the time.

The assumption is then as follows:

(H) The combustion of the propellant obeys the equations

$$\left.\begin{aligned} dx &= r dt \\ \text{and} \quad dV &= S(x) r dt. \end{aligned}\right\} \tag{2}$$

Equations (2) express the increase of the volume V, which is available for gases in the bomb due to the combustion of the propellant. The corresponding change of the unburnt propellant's volume is -dV.

(I) The propellant's surface S(x) is constant.

The last assumption (I) is satisfied, for instance, in the experiments reported in Reference 2. It is essentially a requirement on the geometry of the propellant.

3. OUTLINE OF THE METHOD

Based on the assumptions of Section 2 we may describe the combustion process in the closed bomb by the following set of equations:

$$dE = (\hat{E} - p)\, dV, \tag{3}$$

$$dV = Srdt, \tag{4}$$

$$m = m_o + (V - V_o)\rho_p, \tag{5}$$

where $\hat{E}$ is a constant representing the chemical energy released per unit volume burnt, m_o is the mass of the inert gas present in the bomb at ignition time ($t = o$), ρ_p is the density of the unburnt propellant, and V_o is the initial volume available for the gases. Substitution of (5) into (1) yields a relation between pressure, energy and volume, which is of the form

$$p = E \cdot f(V). \tag{6}$$

Substitution of (6) into (3) yields

$$dE = (\hat{E} - E \cdot f\,(V))\, dV \tag{7}$$

This is a linear differential equation for the internal energy in the closed bomb. Its solution is

$$E(V) = \exp\,[-F(V)] \cdot \left[E_o + \hat{E} \int_{V_o}^{V} \exp\,[F(v)]dv\right], \tag{8}$$

where E_o is the initial value of E, and

$$F(V) = \int_{V_o}^{V} f(v)dv \tag{9}$$

Substituting (8) into (6) we obtain a relation between the pressure and volume in a closed bomb:

$$p(V) = f(V)\, \exp[-F(V)] \left[p_o/f(V_o) + \hat{E} \int_{V_o}^{V} \exp[F(v)]dv\right], \tag{10}$$

where p_o is the initial pressure in the bomb.

This relation is valid for any closed bomb combustion process, which satisfies the assumptions (A) through (G). Note, that the assumptions (H) and (I) about the burning process have not been utilized to derive Equation (10). The equation of state enters Equation (10) through the function f(V).

In some cases, particularly if the quadrature (9) cannot be carried out analytically, it is preferable to use instead of Equation (10) the corresponding differential equation for the pressure. That equation can be derived as follows. Differentiating (6) we obtain

$$dp = f(V)dE + Ef'(V)dV. \tag{11}$$

Next, we eliminate E and dE between (3), (6), and (11), obtaining

$$dp = [f(V)(\hat{E}-p) + pf'(V)/f(V)]dV \tag{12}$$

or

$$\dot{p} = [f(V)(\hat{E}-p) + pf'(V)/f(V)]\dot{V} \tag{13}$$

(By dots we indicate the derivatives with respect to time.) Equation (10) is, of course, the integral of equation (12). Equation (13) may be convenient, if the burning rate formula contains the time derivative $\dot{p}$.

Let the burning rate be expressed by a function containing p, $\dot{p}$ and a parameter vector $\bar{y}$, say,

$$r = G(p,p,\bar{y}) \tag{14}$$

Then, according to the assumptions (H) and (I), we may express the volume by the formula

$$V(t) = V_0 + S\int_0^t r d\tau \tag{15}$$

If p(t) has been observed, we can compute numerically V(t) by (15) for any given set of parameters $\bar{y}$. Therefore, the right hand sides of equations (10) and (13) can be computed, too, as functions of t for any set $\bar{y}$. Since also the left hand sides, p(t) or $\dot{p}(t)$, of those equations have been observed or can be computed from observations, we obtain a curve fitting problem. The constraint is thereby either Equation (10) or Eqaution (13) and the fitting parameters are the components of $\bar{y}$.

This conceptually simple process can be easily extended to equations of state which are more general than Equation (1) and to cases where assumptions (F) and (I) do not hold. However, even under the present restrictions the computations might require non-trival numerical processing, such as extensive tabulation of the functions F(V) and $\int rdt$ for different parameter values and repeated numerical substitutions. The algorithm is simpler if some of the quadratures in Equations (9), (10), and (15) can be carried out analytically. The corresponding formulas are more convenient for discussion. We shall, therefore, derive such formulas for the case of Noble-Abel equation of state with constant specific heats. Since that equation is widely used in interior ballistics, those formulas have also their own merit as working formulas.

4. FORMULAS FOR NOBLE-ABEL GASES

The combustion bomb contains according to our assumptions two gases, namely, an inert gas at the beginning of the experiment to which the combustion gas is added by the burning process. Let both gases obey Noble-Abel equations of state and have constant specific heats. They

can then both be characterized by the following set of parameters:

M - the molar mass,

γ - the ratio of specific heats,

η - the co-volume,

m - the gas mass.

We attach the index "zero" to these quantities, if they refer to initial conditions, that is, to the inert gas, and the index "one", if they refer to the combustion gases. Thus, m_0 is the mass of gas in the bomb at ignition, and $m_1(t)$ (zero at ignition) is the mass of combustion gases. The total gaseous mass in the bomb is

$$m(t) = m_o + m_1(t). \tag{16}$$

The equations of state for the mixture of both gases are the Noble-Abel equation

$$p = RT \frac{m_1/M_1 + m_o/M_o}{V - \eta_1 m_1 - \eta_o m_o} \tag{17}$$

and the energy equation, which is, in case of constant specific heats,

$$E = RT\left(\frac{m_1}{M_1} \cdot \frac{1}{\gamma_1 - 1} + \frac{m_o}{M_o} + \frac{1}{\gamma_o - 1}\right) , \tag{18}$$

where R is the universal gas constant and T is the temperature of the mixture. Eliminating RT between equations (17) and (18), we obtain

$$p = E \cdot \frac{m_1/M_1 + m_o/M_o}{\left(\frac{m_1}{M_1}\frac{1}{\gamma_1 - 1} + \frac{m_o}{M_o}\frac{1}{\gamma_o - 1}\right)\left(V - \eta_1 m_1 - \eta_o m_o\right)} \tag{19}$$

We note, that equation (19) is of the required form (1).

At this point it is convenient to introduce instead of the volume V a new variable z, which represents the volume fraction burnt. Let z be defined by

$$z = (V - V_o)/V_p, \tag{20}$$

where V_o is the initial void volume and V_p is the initial volume of the unburnt propellant. As the combustion progresses, z varies monotonically from zero to one. In terms of z we can express the void volume of the chamber by

$$V = zV_p + V_o \tag{21}$$

and its differential by

$$dV = V_p dz. \tag{22}$$

The gaseous mass m_1 released by the combustion (or the solid mass consumed by the combustion) is in terms of z given by

$$m_1 = zm_p , \tag{23}$$

where m_p is the initial mass of propellant placed in the bomb.

Substituting V and m_1 from Equations (21) and (23) into Equation (19) we obtain after simple algebra

$$p = E \frac{\gamma_1 - 1}{(1-\eta_o\rho_o)V_o} \cdot \frac{z + \frac{m_o}{m_p} \cdot \frac{M_1}{M_o}}{\left(z + \frac{m_o}{m_p} \cdot \frac{M_1}{M_o} \cdot \frac{\gamma_1 - 1}{\gamma_o - 1}\right)\left(z \frac{(1-\eta_1\rho_p)V_p}{(1-\eta_o\rho_o)V_o} + 1\right)} \tag{24}$$

Let

$$\left.\begin{aligned} a &= \frac{(\gamma_1-1)V_p}{(1-\eta_o\rho_o)V_o} \quad ; & b &= \frac{m_o M_1}{m_p M_o} \quad ; \\ c &= \frac{m_o M_1 (\gamma-1)}{m_p M_o (\gamma_o - 1)} \quad ; & d &= \frac{(1-\eta_1\rho_p)V_p}{(1-\eta_o\rho_o)V_o} \quad ; \end{aligned}\right\} \tag{25}$$

and

$$h(z) = a \frac{z + b}{(z+c)\ (zc+1)} . \tag{26}$$

Equation (24) can then be expressed by

$$p = E \cdot h(z) \frac{1}{V_p} \tag{27}$$

The parameters b and c of the function h(z) are positive and generally small compared to one, because usually

$$m_o \ll m_p \quad . \tag{28}$$

The parameter d can be negative as well as positive, depending on the sign of $1 - \eta_1\rho_p$. The only limitation for d is

$$d > -1. \tag{29}$$

This limitation is a consequence of our assumption that the gas mixture obeys Noble-Abel's equation of state (19). That assumption implies that the denominator in Eq. (19) remains positive during the complete combustion. Therefore, we have at ignition the relation:

$$V_o - \eta_o m_o > 0 \tag{30}$$

and after completed combustion the relation

$$V_o + V_p - \eta_o m_o - \eta_1 m_p > 0. \tag{31}$$

Combination of these inequalities yields the inequality (29).

Substituting the expressions (22) and (27) into Eq. (3) we obtain

$$dE = [\hat{E}V_p - Eh(z)]dz. \tag{32}$$

Eq. (32) corresponds to Eq. (7), where the absolute volume V instead of the relative volume z was used. With h(z) given by Eq. (26) the differential equation (32) can be solved analytically except for one quadrature.

The initial value for the solution of the differential equation (32) follows from Eq. (27):

$$E(0) = E_o = p_o \frac{V_p}{h(0)} = p_o \frac{V_p c}{ab} = p_o V_c \frac{1-\eta_o \rho_o}{\gamma_o - 1} . \tag{33}$$

For the analytic solutions of Eq. (32) we have to consider the cases $d = 0$ and $d = 1/c$ separately. In the general case $d \neq 0$ and $d \neq 1/c$ we define

$$\left.\begin{aligned} A &= a \frac{b-c}{1-cd} , \\ B &= \frac{a}{d} \frac{1-bd}{1-cd} . \end{aligned}\right\} \tag{34}$$

The energy E(z) is then given by

$$E(z) = \frac{1}{(z+c)^A (zd+1)^B} \left[c^A E_o + \hat{E}V_p \int_0^z (s+c)^A (sd+1)^B ds \right]. \tag{35}$$

The corresponding pressure p(z) is with Eqs. (26) and (27)

$$p(z) = E(z)\,\frac{a}{V_p}\,\frac{z+b}{(z+c)\,(zd+1)} =$$

$$= \frac{(z+b)a}{(z+c)^{A+1}(zd+1)^{B+1}}\left[\frac{c^{A+1}}{ab}\,p_o + \hat{E}\int_o^z (s+c)^A(sd+1)^B ds\right] \quad (36)$$

Eq. (36) permits to compute the pressure in the chamber as a function of the relative volume consumed. It depends only on the initial pressure and on the gas parameters listed at the beginning of this Section. It is independent of the geometry of the propellant and of the burning rate equations.

The chemical energy constant $\hat{E}$ is equal to the internal energy of the combustion gas at flame temperature, per unit volume of the unburnt propellant. It is for a Noble-Abel gas equal to

$$\hat{E} = \frac{RT_{flame}}{M_1}\,\frac{1}{\gamma_1-1}\,\rho_p = fg\,\frac{1}{\gamma_1-1}\,\rho_p, \quad (37)$$

where f is the "force constant" of the propellant, (Ref.5, p. 96), expressed in metres, and g is the standard acceleration. The dimension of $\hat{E}$ is that of a pressure.

In order to derive an equation corresponding to Eq. (13) we follow the steps outlined in Section 3, obtaining first the equation

$$\dot{p} = [(\hat{E}-p)h(z) + ph'(z)\,/h(z)]\dot{z}. \quad (38)$$

With h(z) from Eq.(26) we compute the derivative

$$h'(z) = -a\frac{(z+b)^2 d + (b-c)(1-bd)}{(z+c)^2(zd+1)^2} \quad (39)$$

and substitute h(z) and h'(z) into Eq.(38). The result is

$$\dot{p} = \left[(\hat{E}-p)\,\frac{z(z+b)}{(z+c)(zd+1)} - p\,\frac{(z+b)^2 d + (b-c)(1-bd)}{(z+b)(z+c)(zd+1)}\right]\dot{z} \quad (40)$$

Eq.(36) gives the pressure p as a function of the relative volume consumed. In cases of practical interest p(z) is strictly monotonic function and can be inverted to yield z as a function of p. By substitution of that inverted function into Eq.(40) one obtains an expression of the type

$$\dot{p} = C(p)\cdot\dot{z}\ , \quad (41)$$

[5] Corner, F., "Theory of the Interior Ballistics of Guns" John Wiley and Sons, New York, 1950.

where C(p) is independent of the burning rate and of the burning surface conditions. This equation is a central equation for the numerical evaluation of closed bomb measurements. Usually it is assumed that C(p) can be approximated by a constant, namely, the final pressure in the bomb, p_{end} (Ref. 1-4). The equation

$$\dot{p} = p_{end} \cdot \dot{z} \tag{42}$$

can be derived theoretically if one assumes uniform gas, constant temperature, negligible volume change, $p_o << p_{end}$, and $\eta_1 \rho_p = 1$. Another approximation,

$$\dot{p} = p_{end} \dot{z} + b', \tag{43}$$

where b' is an experimental fitting parameter, is used in Refernces 2 and 3.

In our analysis C(p) is controlled by the setup of the experiment. Because a constant C(p) greatly facilitates the processing of the data, it is desirable to plan the experiments such, that a constant is a good approximation to C(p). In the next Section we shall show by some examples how C(p) depends on various parameters of the experiment.

The fitting of experimental data with the aid of Eq.(41) can be best illustrated by an example. Let the burning rate be given by the equation

$$r = \beta p^n + \frac{\alpha}{\beta} \frac{n}{p^{n+1}} \dot{p}, \tag{44}$$

where β and n are free parameters and α is the thermal diffusivity of the unburnt solid propellant. The first term, βp^n, on the right hand side of Eq. (44) is the usual form of a "steady burning rate equation". The benefits of adding the second ("transient")term are discussed at length in Reference 3. For some propellants such a term results in a more realistic description of the burning behavior, particularly for low pressures. The time derivative $\dot{z}$ is given with Eqs. (15) and (20) in this case by

$$\dot{z} = \frac{S}{V_p} \left[\beta p^n + \frac{\alpha}{\beta} \frac{n}{p^{n+1}} \dot{p} \right]. \tag{45}$$

Substitution of this expression into Eq. (41) yields a differential equation for p(t), namely,

$$\dot{p} = C(p) \frac{S}{V_p} \left[\beta p^n + \frac{\alpha}{\beta} \frac{n}{p^{n+1}} \cdot \dot{p} \right]. \tag{46}$$

Obviously Eq. (46) can be integrated analytically if C(p) is constant. The integral is an equation containing p,t and the unknown parameters

β and n. That equation can be used as a constraint functional for fitting the observed p(t).

For sake of completeness we note here also the final formulas for the special cases excluded above.

First we consider the case d=0, that is, $\underline{\eta_1 \rho_p = 1}$. The pressure function p(z) is in this case

$$p(z) = \frac{(z+b)ae^{-az}}{(z+c)^{(b-c)a+1}} \left[p_o \frac{c^{(b-c)a+1}}{ab} + E \int_o^z (s+c)^{(b-c)a} e^{as} ds \right] . \quad (47)$$

The relation between $\dot{p}$ and $\dot{z}$, corresponding to Eq. (40), is

$$\dot{p} = [\hat{E}a + (c-b-a)p] \frac{z+b}{z+c} \dot{z}. \quad (48)$$

In the special case $\underline{d=1/c}$ we obtain the corresponding equations

$$p(z) = \frac{(z+b)ac^{1+ac}}{(z+c)^{2+ac}} \exp\left(\frac{(c-b)az}{z+c}\right)\left[\frac{cp_o}{ab} + \hat{E} \int_o^z \left(1+\frac{s}{c}\right)^{ac} \exp\left(\frac{(b-c)as}{s+c}\right) ds\right] (49)$$

and

$$\dot{p} = \left[(\hat{E}-p) \frac{(z+b)ac}{(z+c)^2} - p \frac{(z+b) - (b-c)^2}{(z+b)(z+c)^2}\right] \dot{z} . \quad (50)$$

5. DISCUSSION OF A SPECIAL CASE

The formulas derived in the previous section still contain integrals which must be evaluated numerically in order to compute the function C(p). In this section we shall discuss a special case which produces simplifications of those formulas. The purpose of the discussion is to provide a general picture of the properties of C(p). Particularly we are interested to find conditions under which C(p) can be approximated by a constant.

A common assumption for the treatment of closed bomb problems is to assume ideal gases ($\eta_0 = \eta_1 = 0$). In our analysis such an assumption does not simplify any expressions.

Inspecting the equations of the previous section we observe that considerably simpler formulas can be obtained if the parameters b and c of the function h(z) can either be neglected or are equal. The latter is the case, for instance, if the inert gas and the combustion gas are identical. However, in order to make b=c, it is sufficient to require only

$$\gamma_0 = \gamma_1 . \quad (51)$$

We assume for the rest of this Section, that Eq.(51) holds. The function h(z) is then simply given by

$$h(z) = \frac{a}{zd+1} \tag{52}$$

and the quadratures of Section 4 can be carried out analytically. Thereby we have to consider the special cases $\eta_1\rho_p = 1$ and $\eta_1\rho_p = \gamma_1$ separately.

In the general case, that is, for $\eta_1\rho_p \neq 1$, $\nu_1\rho_p \neq \gamma_1$, and $\gamma_o = \gamma_1$ we obtain the following formulas

$$\begin{aligned} p(z) &= \hat{E}\,\frac{B}{B+1} + \left[p_o - \hat{E}\,\frac{B}{B+1}\right] \frac{1}{(zd+1)^{B+1}} = \\ &= \hat{E}\,\frac{B}{B+1} + \left[p_{end} - \hat{E}\,\frac{B}{B+1}\right] \left(\frac{d+1}{zd+1}\right)^{B+1}, \end{aligned} \tag{53}$$

$$\begin{aligned} C(p) &= \left[B\hat{E} - (B+1)p_o\right] \cdot d \cdot \left[\frac{B\hat{E} - (B+1)p}{B\hat{E} - (B+1)p_o}\right]^{(B+2)/(B+1)} = \\ &= \left[B\hat{E} - (B+1)p_{end}\right] \cdot \frac{d}{d+1} \cdot \left[\frac{B\hat{E} - (B+1)p}{B\hat{E} - (B+1)p_{end}}\right]^{(B+2)/(B+1)}. \end{aligned} \tag{54}$$

In the special case $\eta_1\rho_p = \gamma_1$ and $\gamma_o = \gamma_1$ we have

$$\begin{aligned} p(z) &= p_o - \hat{E} \cdot \ln(zd+1) = \\ &= p_{end} - \hat{E} \cdot \ln\left(\frac{zd+1}{d+1}\right) \end{aligned} \tag{55}$$

and

$$\begin{aligned} C(p) &= -\hat{E} \cdot d \cdot \exp\left(\frac{p-p_o}{\hat{E}}\right) = \\ &= -\frac{d}{d+1} \cdot \hat{E} \cdot \exp\left(\frac{p-p_{end}}{\hat{E}}\right). \end{aligned} \tag{56}$$

In the case $\eta_1\rho_p = 1$ and $\gamma_o = \gamma_1$ the formulas are

$$\begin{aligned} p &= \hat{E} - (\hat{E} - p_o)\exp(-az) = \\ &= \hat{E} - (\hat{E} - p_{end})\exp[(1-z)a] \end{aligned} \tag{57}$$

and

$$C(p) = a \cdot (\hat{E} - p). \tag{58}$$

Typical examples of the functions p(z), C(p) and of the temperature within the bomb are shown in Figures 1 through 3. They illustrate the dependence of these functions on some parameters of the closed bomb experiment. The basic parameter values used for these calculations are listed in Table 1.

The amount of propellant enters the analysis in form of the volume ratio V_p/V_o (see Eq.(25)). A more convenient measure for the amount of propellant is the ratio of the propellant's volume to the volume of the bomb. That ratio is chosen as the variable parameter for the curves in Figure 1. It is related to V_p/V_o by the equation

$$\frac{V_p}{V_o} = \frac{V_p}{V_{bomb}} \cdot \frac{1}{1-V_p/V_{bomb}}. \tag{59}$$

The examples shown in Figure 1 indicate, that an approximation of C(p) by a constant is feasible in the present case only for loading volume ratios V_o/V_{bomb} less than 0.1. For these cases and about one atmosphere (10^5Pa) initial pressure $C^* = p_{end}$ is a reasonable approximation to C(p). The corresponding temperature curves T(p), are almost constant over most of the pressure range, if $V_p/V_{bomb} > 0.1$. For high loading volume ratios the temperature in the bomb reaches a maximum at lower pressures and then decreases, as the combustion progresses.

The examples in Figure 2 show the dependence of the functions C(p) and T(p) on the inital pressure p_o. It is interesting to note that for small loading volume ratios ($V_p/V_{bomb} = 0.05$ in Figure 2) the function C(p) can be best approximated by a constant which is less than p_{end}.

Figure 3 shows the significance of the co-volume term for the analysis. The special cases $\eta_1 = 1/\rho_p$ and $\eta_1 = \gamma/\rho_p$ exhibit no special features in these examples.

An inspection of the formulas of this section shows that in general C(p) can be approximated by a constant if $p \ll \hat{E}$. The value of $\hat{E}$ depends on the physical parameters of the propellant only. The maximum value of the pressure depends on the other hand also on the ratio V_p/V_o as well as on p_o. Therefore, the condition $p \ll \hat{E}$ limits for a given propellant the maximum admissible ratio V_p/V_o and the maximum initial pressure p_o.

The examples presented indicate also that the behavior of the temperature in the bomb is by itself not relevant to the question whether C(p) can be approximated by a constant.

Table 1

Basic Parameter Values for Figures 1 Through 3

Inert gas	Combustion gas
$\gamma_o = 1.21$	$\gamma_1 = 1.21$
$\eta_o = 1.375 \cdot 10^{-3}\ m^3/kg$	$\eta_1 = 1.09 \cdot 10^{-3}\ m^3/kg$
$M_o = 28 \cdot 10^{-3}$ kg/mol	$f \cdot g = 1.186 \cdot 10^6\ m^2/s^2$
$T_o = 300$ K	$T_{flame} = 3700$ K
$p_o = 10^5 Pa$	$\rho_p = 1.62 \cdot 10^3\ kg/m^3$
	$(\hat{E} = 9.15 \cdot 10^9\ Pa)$

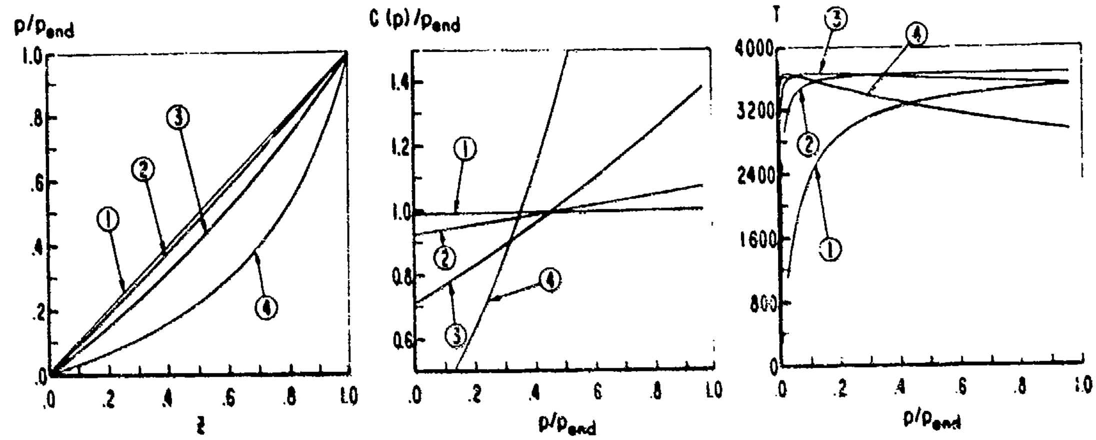

Figure 1. Closed Bomb Functions for Different Loading Volumes

Fixed Parameters: $p_o = 10^5$ Pa

$\eta_1 = 10^{-3}\ m^3/kg$

Variable Parameter: $V_p/V_{BOMB} = 0.01; 0.1; 0.3; 0.5$

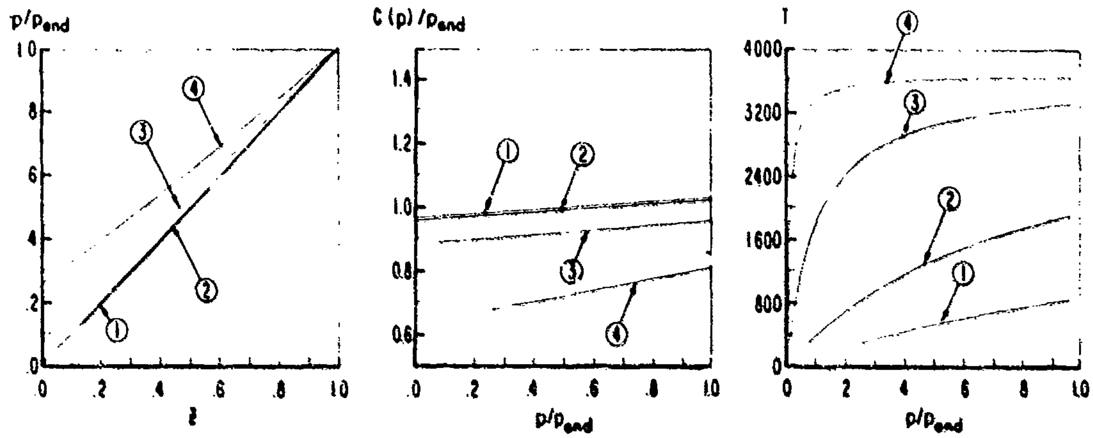

Figure 2. Closed Bomb Functions for Different Initial Pressures

Fixed Parameters: $V_p / V_{BOMB} = 0.05$

$\eta_1 = 10^{-3}\ m^3/kg$

Variable Parameter: $p_0 = 10^5; 10^6; 10^7; 10^8$ (Pa)

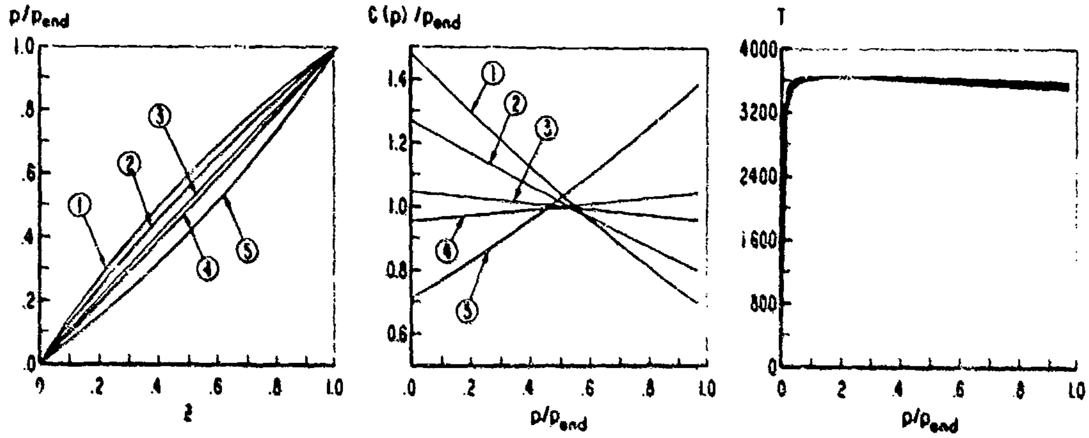

Figure 3. Closed Bomb Functions for Different Co-volumes

Fixed Parameters: $V_p/V_{BOMB} = 0.3$

$p_0 = 10^5$ Pa

Variable Parameter: $\eta_1 = 0;\ 0.3 \cdot 10^{-3};\ 1/\rho_p;\ \gamma_\cdot/\rho_p;\ 10^{-3}\ (m^3/kg)$

6. EXAMPLE

In this Section we shall give an example for the application of the closed bomb data reduction algorithm. The data are taken from two recent experiments at the Ballistics Research Laboratories (Ref. 6). The important parameters of the experiments are listed in Table 2. Because in this case $\gamma_0 = \gamma_1$, the simple formulas of Section 5 could be used to compute the function C(p). The result shown in Figure 4 indicates that C(p) is within 10% of the end pressure throughout the combustion process. However, the end pressure computed by Eq. (53) was bigger than the observed end pressure. It was conjectured that the reason for this discrepancy could be an overestimation of the propellants "force" constant. Assuming therefore a 34.8% smaller "force" constant both pressures were matched. The corresponding new C(p)-curve was found to be within plotting accuracy of Figure 4 equal to the curve computed for the higher "force" constant, although the corresponding temperatures in the bomb differed by up to 1200K. It was then concluded that, independently of the true value of the "force" constant, the observed p_{end} is a reasonable approximation to C(p).

Let C* be a constant approximation to C(p). The integrated constraint equation (46) is then for $n \neq 1$

$$F(t,p;\beta,n) = \frac{V_p}{C^*S} \cdot \frac{1}{\beta} \cdot \frac{1}{1-n} \left(p^{1-n} - p_o^{\,1-n}\right) + \tag{60}$$
$$+ \frac{\alpha}{2\beta^2} \left(p^{-2n} - p_o^{\,-2n}\right) - t + t_o = 0,$$

and for n = 1

$$F(t,p;\beta,1) = \frac{V_p}{C^*S} \cdot \frac{1}{\beta} \cdot \ln\,(p/p_o) + \tag{61}$$
$$+ \frac{\alpha}{2\beta^2} \left(p^{-2} - p_o^{\,-2}\right) - t + t_o = 0 .$$

In Eqs. (60 and (61) t_o and p_o is a pair of corresponding values of observed time and pressure, respectively.

In the present case the terms containing the thermal diffusivity α were found to be insignificant and were therefore not used in the final evaluations. Also, the following more general steady state burning rate formula was used for the analysis:

$$r = \beta(p - \hat{p})^n. \tag{62}$$

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[6]Knapton, J.D., Interior Ballistics Laboratory, USA Ballistic Research Laboratories, private communication. The experiments have IBL numbers 167-3 and 167-4, dated 21 March 1974. The propellant sample was JPN, coated with a polystyrene surface inhibitor.

Table 2

Parameters of the Experiment

$$V_o/V_{bomb} = 0.1278$$

$$C^* = p_{end} = 2.04 \cdot 10^8 \text{ Pa}$$

$$V_p/S = 6.4922 \cdot 10^{-2} \text{ m}$$

Inert gas	Combustion gas
$\gamma_0 = 1.21$	$\gamma_1 = 1.21$
$\eta_0 = 1.375 \cdot 10^{-3}$ m^3/kg	$\eta_1 = 1.093 \cdot 10^{-3}$ m^3/kg
$M_0 = 28 \cdot 10^{-3}$ kg/mol	$M_1 = 25.66 \cdot 10^{-3}$ kg/mol
$T_0 = 300$ K	$\rho_p = 1.62 \cdot 10^3$ kg/m^3
$p_0 = 1.013 \cdot 10^5$ Pa	Given $fg = 1.186 \cdot 10^6$ m^2/s^2
	Used $fg = 0.7738 \cdot 10^6$ m^2/s^2
	($\hat{E} = 5.970 \cdot 10^9$ Pa)

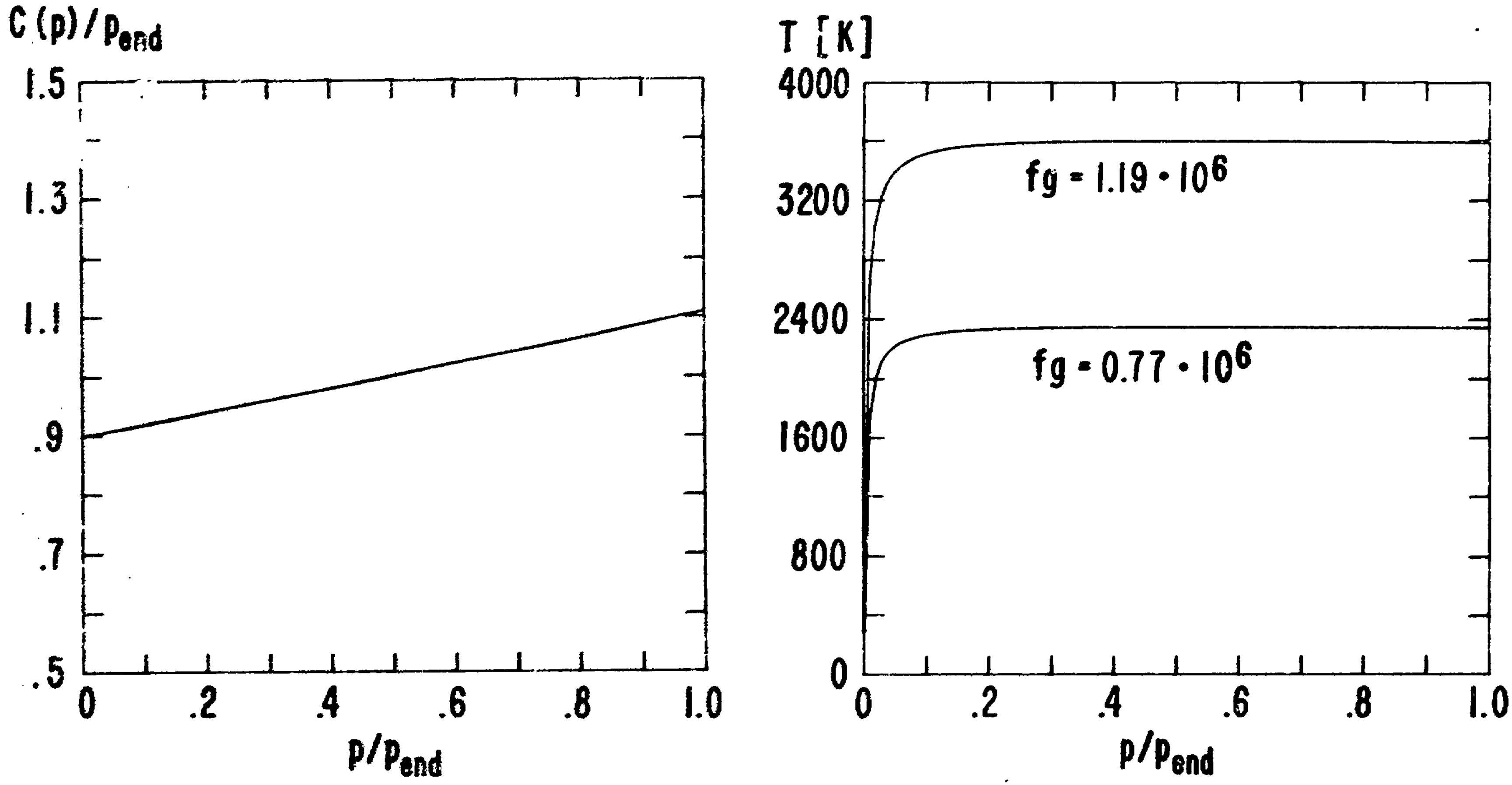


Figure 4. Closed Bomb Function C (p) and Computed Temperatures for the Experiments.

With this burning rate formula the integrated constraint functional is for $n \neq 1$

$$F(t,p;\beta,n,\hat{p}) = \frac{V_p}{C^*S}\,\frac{1}{\beta}\,\frac{1}{1-n}\,\left[(p-\hat{p})^{1-n} - (p_o-\hat{p})^{1-n}\right] - t + t_o = 0 \quad (63)$$

and for $n = 1$

$$F(t,p;\beta,1,\hat{p}) = \frac{V_p}{C^*S}\,\frac{1}{\beta}\,\ln\frac{p-\hat{p}}{p_o-\hat{p}} - t + t_o = 0\,. \quad (64)$$

The equations (63) and (64) define a three parameter family of curves in the t,p-plane. They all pass through the node (t_o,p_o), which corresponds to an arbitrary integration constant. That constant is an additional parameter of the curves and must be included in the curve fitting process. In the present case it is convenient to chose an arbitrary p_o and treat t_o as a free parameter. The adjustment problem has then a total of four parameters: $\beta,n,\hat{p}$, and t_o.

The pressure data were obtained simultaneously from two pressure transducers (Figure 5). Consequently for each time value two pressure values were recorded. These double recordings were used to assess the accuracy of the measurements. By inspection of the data scattering and of the discrepancies between simultaneous pressure readings it was estimated that the standard error of each pressure reading was approximately

$$e_p = 2 \cdot 10^6 + 0.02p \text{ [Pa]}. \quad (65)$$

The standard errors of the time observations were assumed to be

$$e_t = 0.1 \text{ [ms]}. \quad (66)$$

Correlations between pressure and time readings were assumed to be negligible.

For the data reduction example pressure readings were sampled from a more detailed list at 0.5ms or 1.0ms intervals. Figure 6 shows the sampled pressure-time data from two experiments together with some 3e-error ellipses, indicating the accuracy of the data. High pressure measurements appear in Figure 6 to be more accurate because the pressure is plotted in logarithmic scale. In fact only the relative accuracy is better at high pressures.

The rapid pressure increase shown in Figure 6 occured in the experiments about 1.5s after ignition. Pressure readings prior to that increase have a very high noise level and cannot be used without special treatment. In the present evaluation examples we did not use those observations at all. The complete p(t) curve has an appearance which is schematically shown in Figure 7. Only a small part of the curve for $t \gtrsim 1.5$s is well

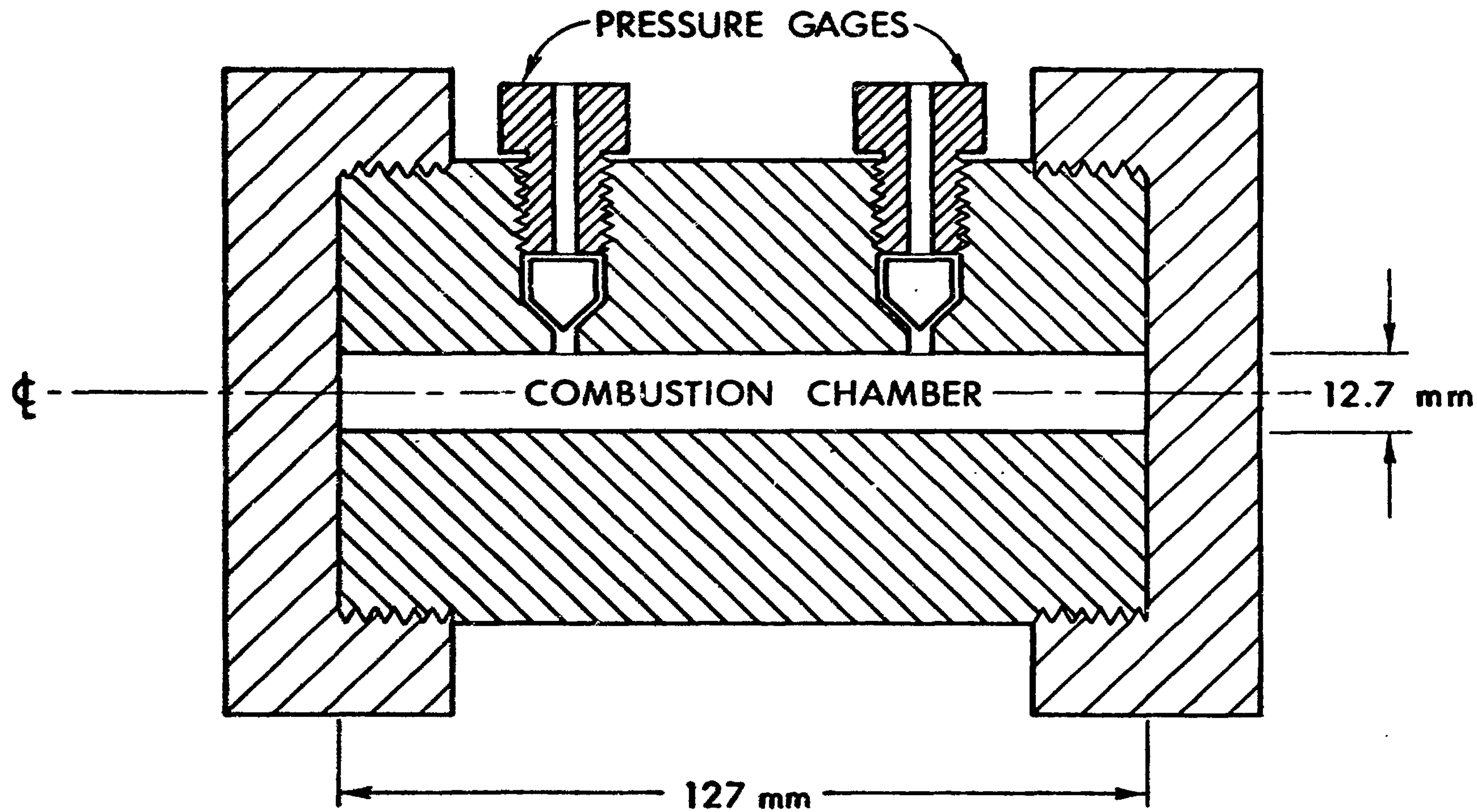
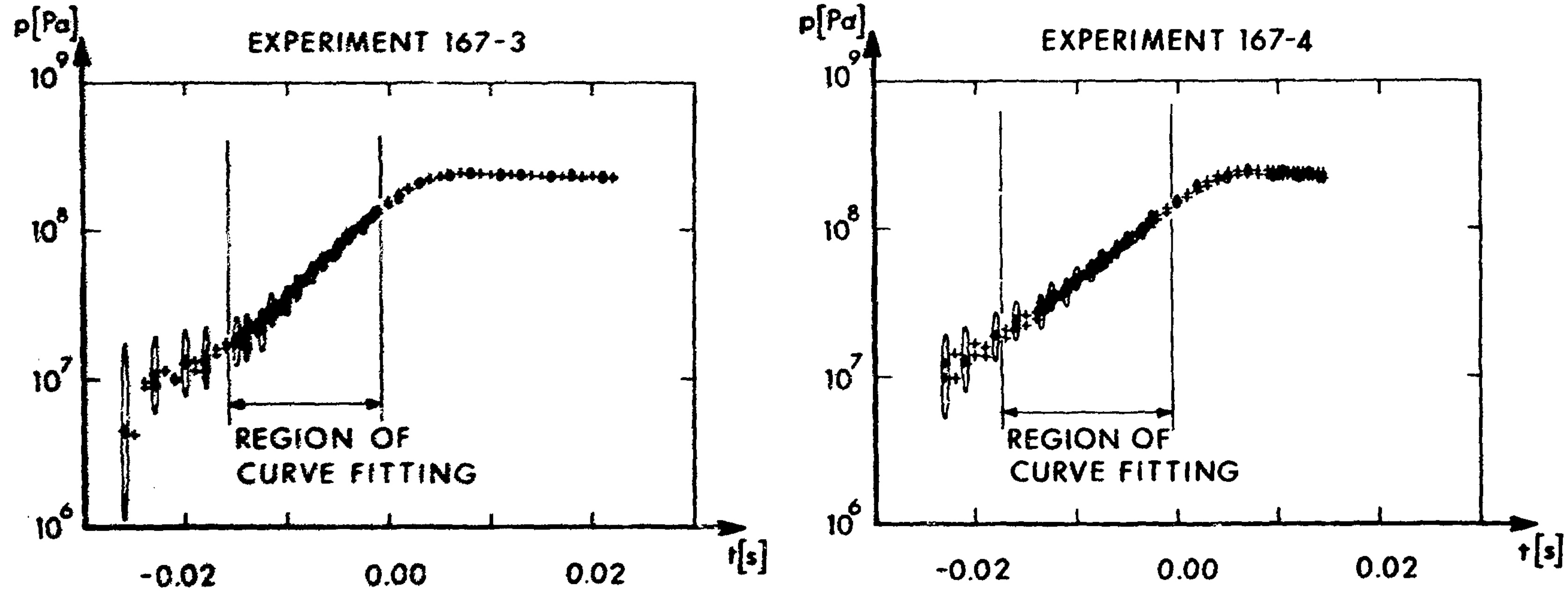


Figure 5. Closed Bomb for Burning Rate Measurements.



NOTE: $t = 0$ IS AN ARBITRARY REFERENCE TIME

Figure 6. Observed Pressure Histories

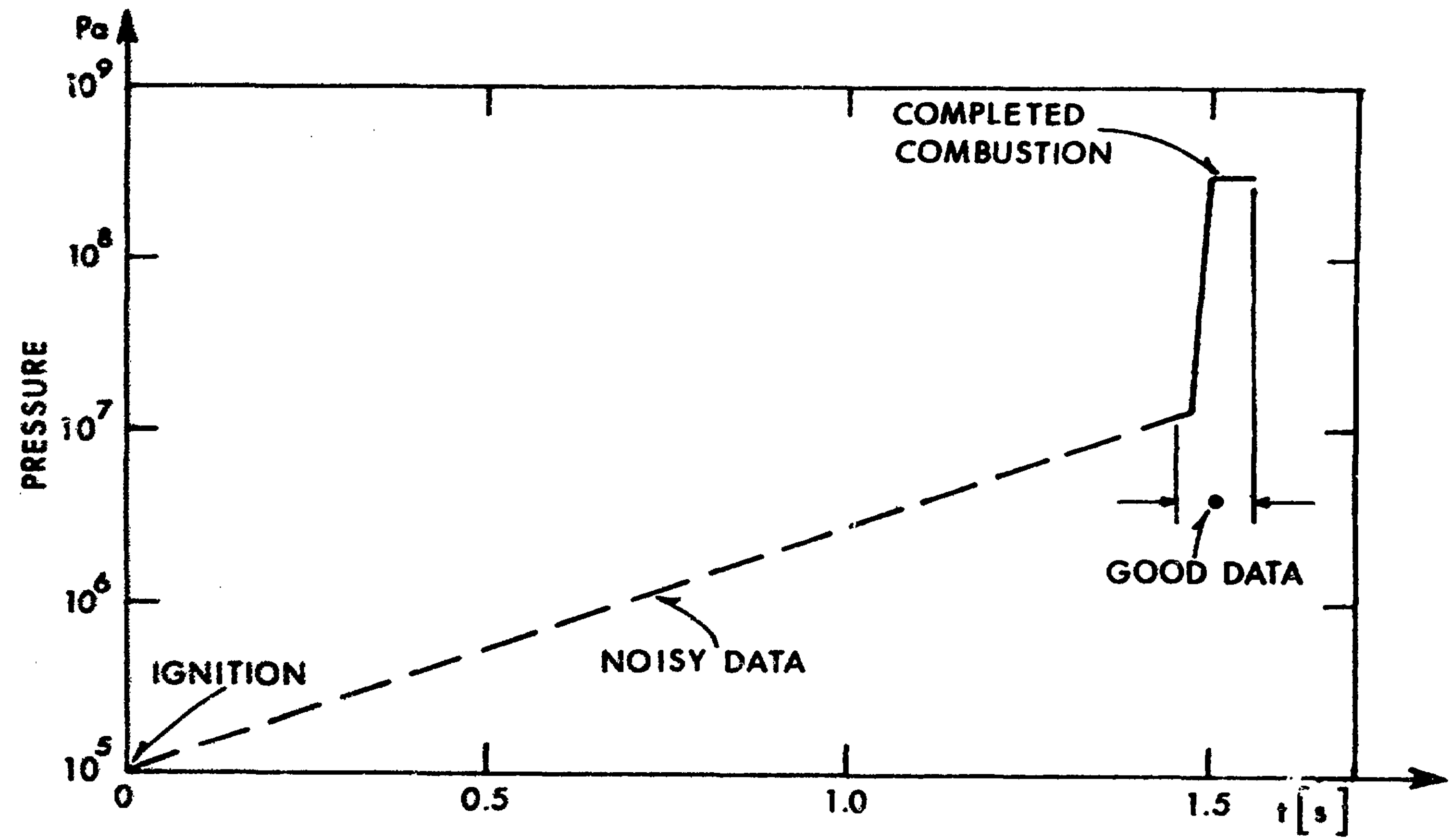


Figure 7 Schematic Pressure History In Experiments 167-3 And 167-4

defined by measurements. The time coordinate of the ignition point is known only approximately and data between t = 0 and t = 1.5 are practically not available.

The constant part of the curve (after completed combustion) is not described by eqs. (63) and (64). The corresponding data should, therefore, not be used for data reduction (except for determining p_{end}). On the other hand the transition from slow to fast pressure increase at about 1.5 s is not modeled either by the family of curves defined by eqs. (63) and (64). It seems that in the experiment two different burning processes take place: one for pressures, say, below 20MPa and a different one for higher pressures. Under these circumstances low pressure observations and high pressure observations should be evaluated separately. We have therefore excluded from our analysis those observations which are obviously within the slow pressure increase region or within the transition between slow and fast increase regions. The choice was made by visual inspection of Figure 6.

The restriction of data to the interval between about 20MPa and 200MPa means that any burning laws computed from those data are valid within that pressure region only.

Examples of curve fitting using eqs. (63) and (64) are shown in Figures 8 and 9. The adjustment was obtained by the non-linear least squares algorithm described in Reference 7. The data points which were used for the curve fitting are marked in Figures 8 and 9 by black squares. From each experiment 58 such points were sampled at 0.5 ms time intervals within the fast pressure increase region. The figures show the fitted curves together with their three standard error confidence limits for different burning rate formulas, namely,

$$r = \beta p,\ r = \beta p^n,\ r = \beta(p - \hat{p}),\ \text{and}\ r = \beta(p - \hat{p})^n.$$

It appears from the plots that the curve fitting does not improve if two - and three-parameter burning rate functions are used instead of the simple linear function $r = \beta p$. The numerical results are summarized in Table 3 and confirm that impression. The values of W (the weighted sum of correction squares) and e_0 (the square root of variance of weight one) are affected only insignificantly by the addition of parameters.

The reasons for this result become clear if one considers the magnitude of the correlation coefficients between the parameters. They are all very close to one, particularly the correlation coefficients between β and n. This indicates that all parameters affect the fitting curve in a similar manner within the range of observations.

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[7]Celmiņš, A., "Least Squares Adjustment with Finite Residuals for Non-Linear Constraints and Partially Correlated Data", Ballistic Research Laboratories Report No. 1658, July 1973. (AD #766283)

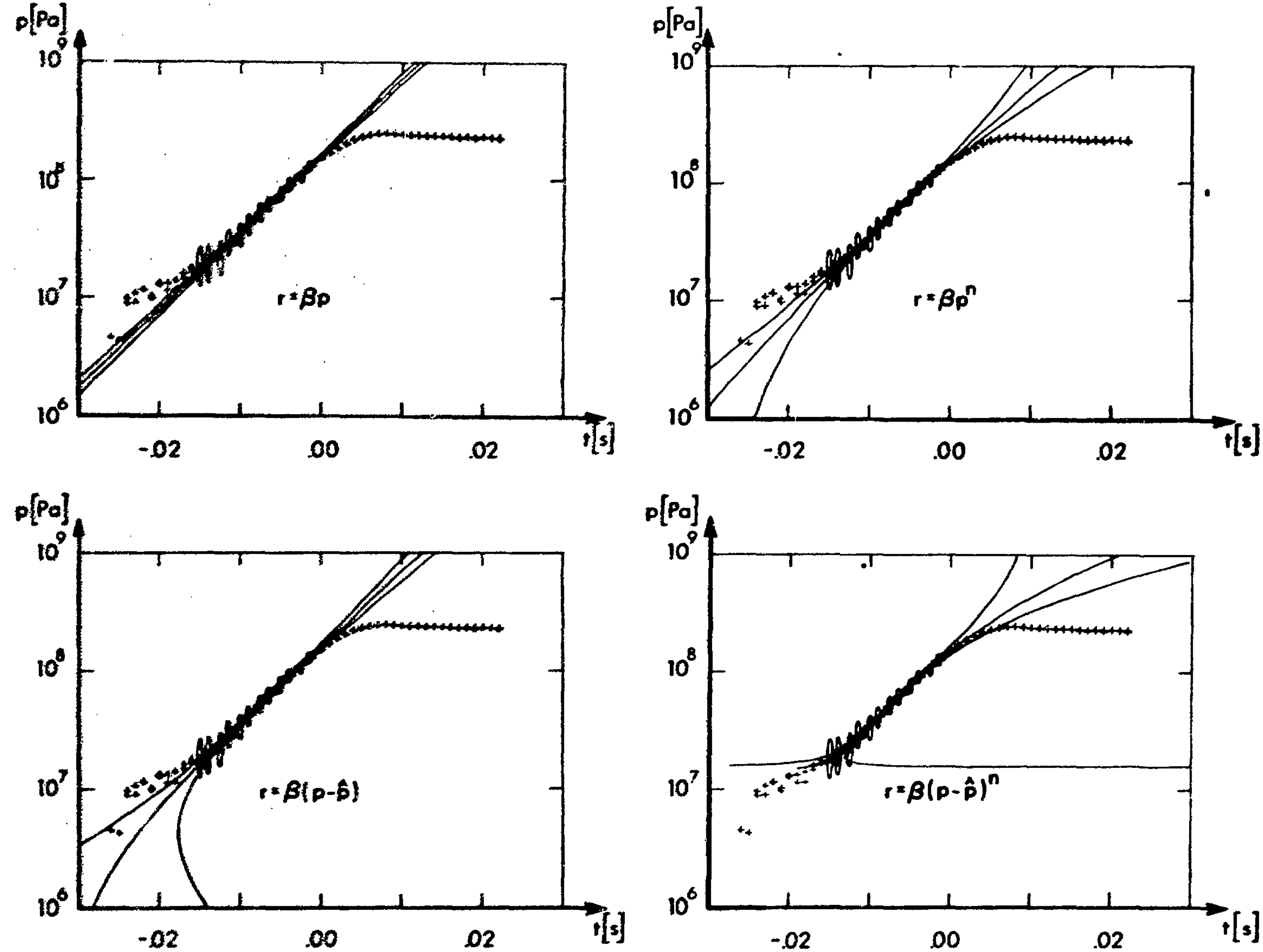

Figure 8. Curve Fitting To Data Of Experiment 167-3

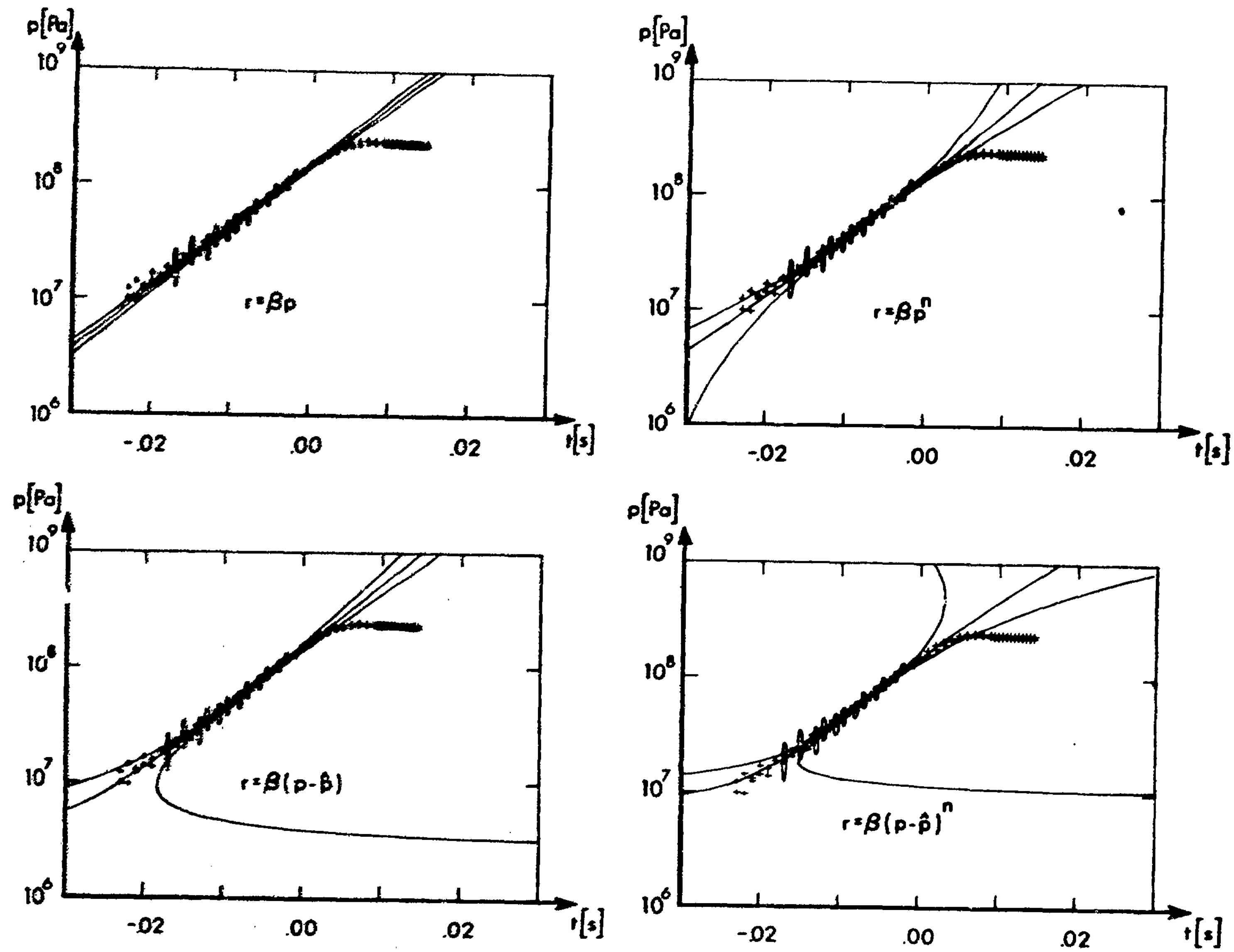

Figure 9. Curve Fitting To Data Of Experiment 167-4

Experiment	$\beta[m \cdot s^{-1} \cdot Pa^{-n}]$	n	$\hat{p}$[MPa]	W	e_o	Parameter correlation coefficients (β,n)	$(\beta,\hat{p})$	$(n,\hat{p})$
167-3	$(3.761 \pm 0.041) \cdot 10^{-8}$	1.0	0.0	32.40	0.761	---	---	---
167-4	$(5.192 \pm 0.036) \cdot 10^{-8}$	1.0	0.0	27.52	0.701	---	---	---
167-3	$(9.02 \pm 5.92) \cdot 10^{-8}$	0.951 ± 0.037	0.0	31.44	0.759	-0.9999	---	---
167-4	$(1.54 \pm 1.18) \cdot 10^{-8}$	1.040 ± 0.043	0.0	27.09	0.701	-0.9999	---	---
167-3	$(3.650 \pm 0.121) \cdot 10^{-8}$	1.0	-1.60 ± 1.69	32.01	0.762	---	+0.9430	---
167-4	$(3.336 \pm 0.127) \cdot 10^{-8}$	1.0	2.37 ± 1.93	26.92	0.700	---	+0.9553	---
167-3	$(33.63 \pm 51.49) \cdot 10^{-6}$	0.633 ± 0.084	15.12 ± 3.24	27.76	0.717	-0.9999	+0.9481	-0.9438
167-4	$(0.4603 \pm 1.3485) \cdot 10^{-6}$	0.860 ± 0.157	8.95 ± 7.29	26.63	0.702	-0.9999	+0.9736	-0.9710
JPN in Ref. 8 for 70°F	$0.2868 \cdot 10^{-6}$	0.69	0.0	---	---	---	---	---

Table 3. Least Squares Values and Standard Errors of Parameters in the Burning Rate Formula $r = \beta(p - \hat{p})^n$.

In Figure 10 we show the least squares values of β and n in a n,lgβ plane. The strong correlation between these parameters let the error ellipses look like inclined error bars. The JPN parameter values of Ref. 8 are plotted in Figure 10 for comparison.

The differences between burning rates computed using the various burning rate formulas are not as large as it appears considering the values of the parameters only. If the burning rate is plotted as a function of pressure, then all burning rate formulas produce comparable burning rate values withing the range of observations. (See Figures 11a through 11d). The main differences between the various burning laws are their different standard errors. The errors are larger, for formulas with more parameters, because the simplest curve ($r = \beta p$) is already located well within the data scattering.

The similarity of the burning laws is demonstrated also in Figure 12, where the various burning rate functions of Table 3 are plotted in one graph. The eight curves corresponding to the first eight burning rate functions of Table 3 are close to each other within the pressure interval of observations. The burning rate function of Reference 8 differs from the other functions by a factor of 25 to 50 within the same pressure interval. The reason for this difference is possibly the fact that Reference 8 deals with burning rates at low pressures.

In summary, these results indicate that the simplest burning rate formula $r = \beta p$ is a good approximation for JPN at pressures between 20 MPa and 200MPa. The coefficient β can be computed as a weighted average of the corresponding values of both experiments. The final average burning rate formula is then

$$r = (3.431 \pm 0.029)\ 10^{-8}\ p, \tag{65}$$

where r is expressed in m/s and p in Pa.

7. CONCLUSIONS AND RECOMMENDATIONS

The closed bomb theory presented in this report permits one to establish rational limits for the validity of simplified data reduction formulas which are presently in use. Expansion of the theory to more general cases, including for instance variable surface propellants and variable C(p) is conceptually simple, and requires merely the development of complicated computer codes.

The data reduction process has been formulated as a nonlinear curve fitting task. Such an approach permits one to apply standard curve-fitting theory and algorithms to the problem, whereby valuable information about the accuracies and correlations of the burning rate parameters can be obtained. It was found that in general these parameters are strongly

[8] Couch, S., "Some Properties of Several Double Base Solid Propellants for Rockets", NAVORD Report #3477, August 1955.

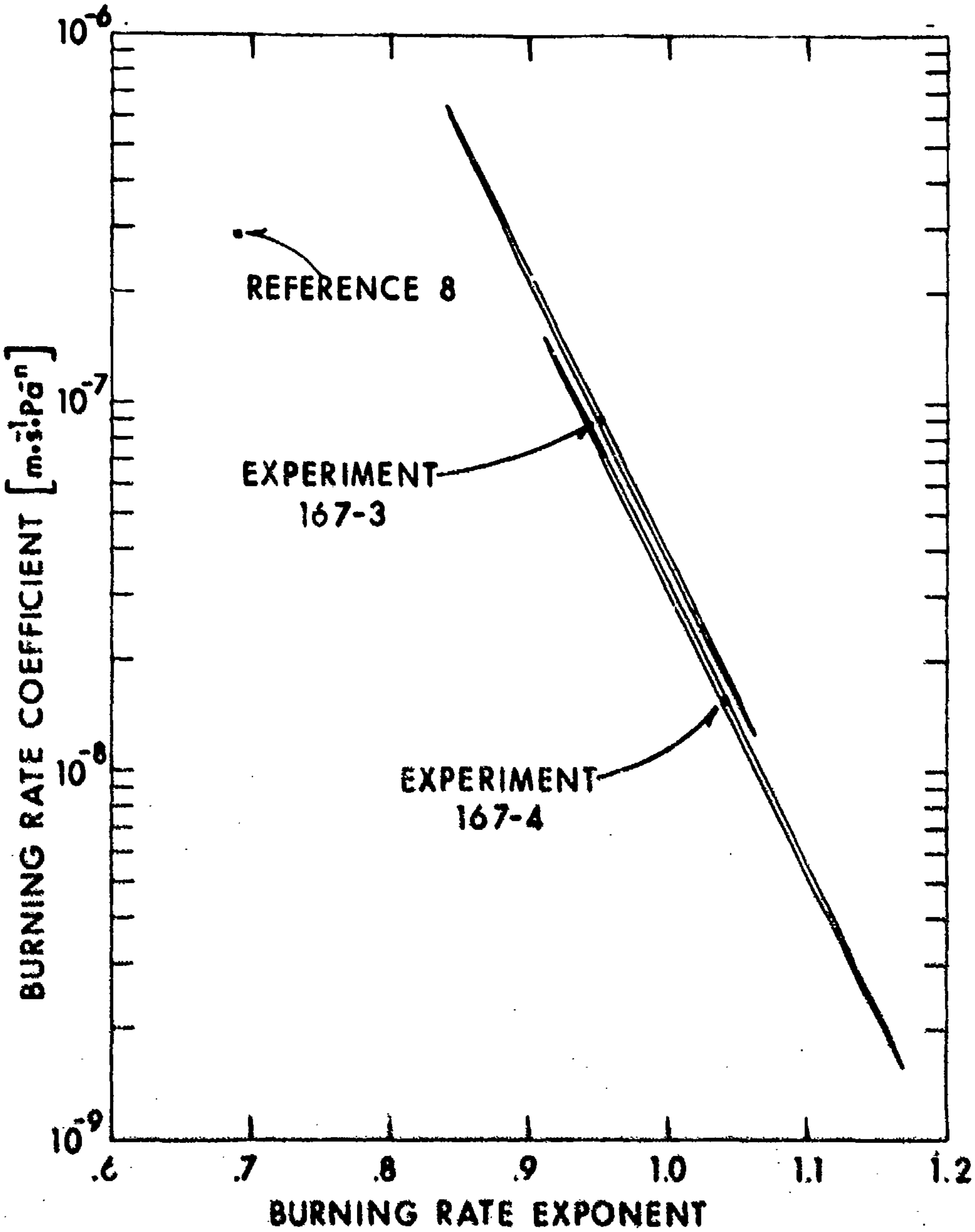

Figure 10 Least Squares Values And Error Ellipses Of Burning Rate Parameters

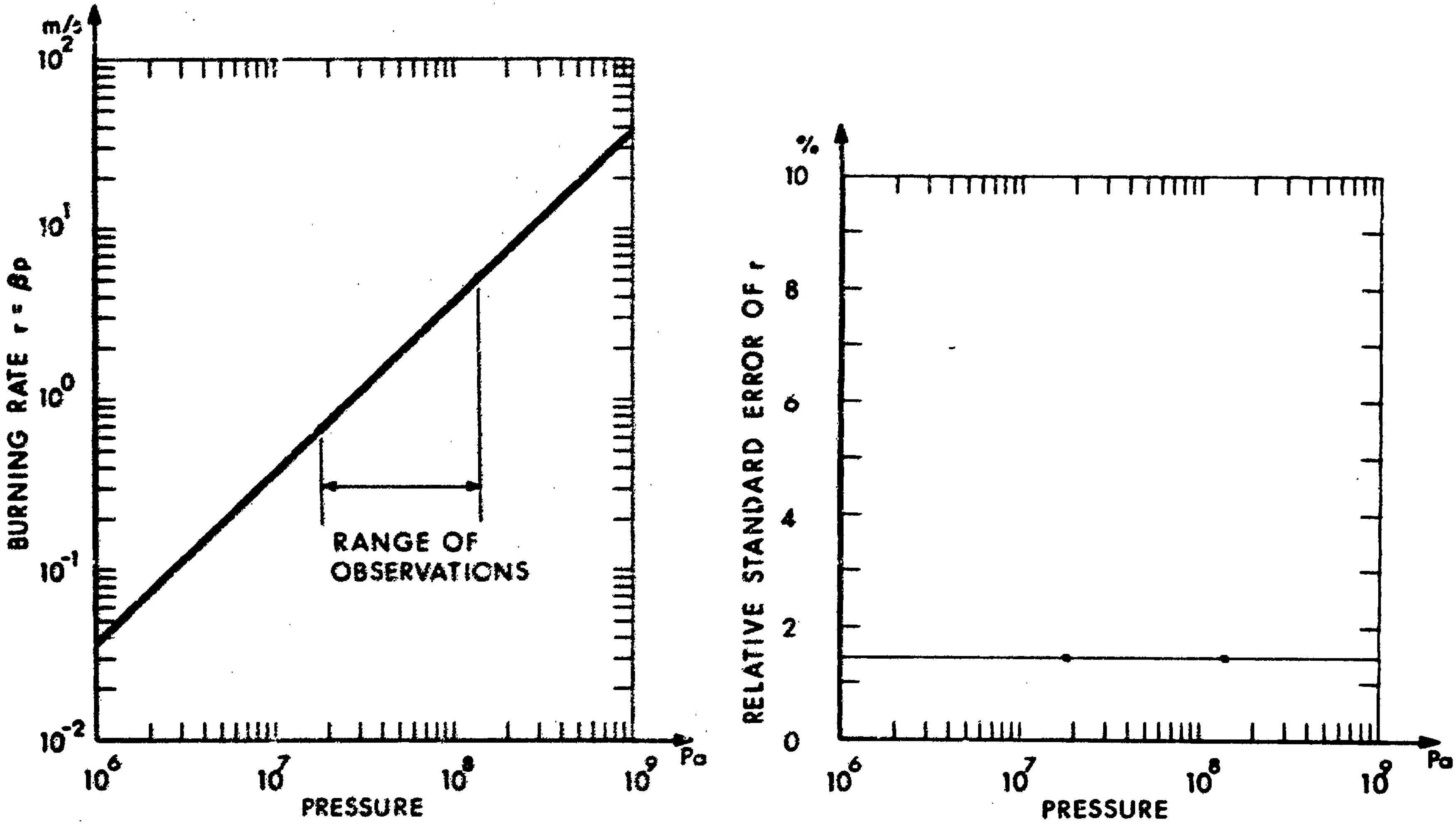

Figure 11a. Burning Rate $r = \beta p$ From Experiment 167-3

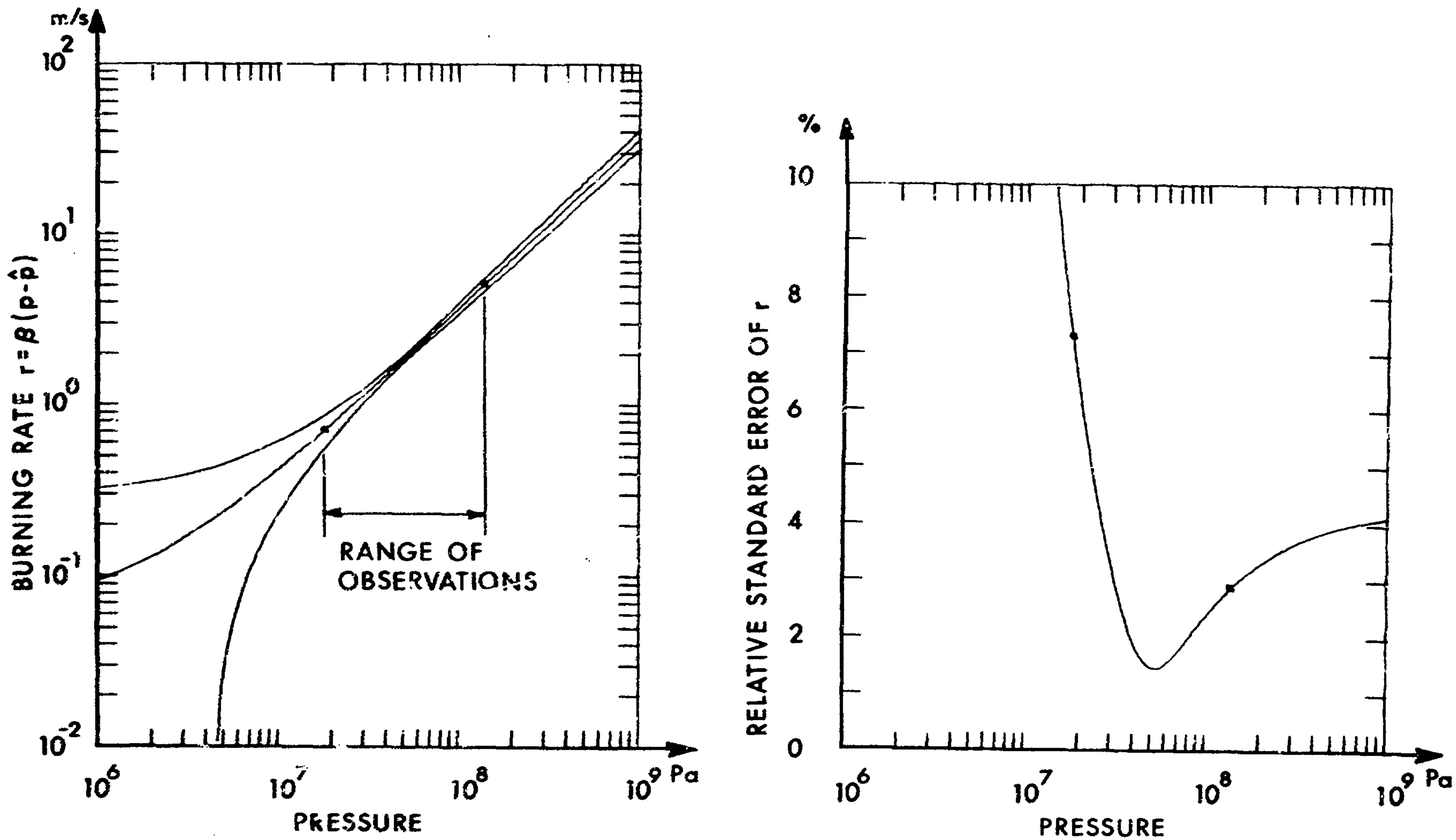

Figure 11b. Burning Rate $r=\beta(p-\hat{p})$ From Experiment 167-3

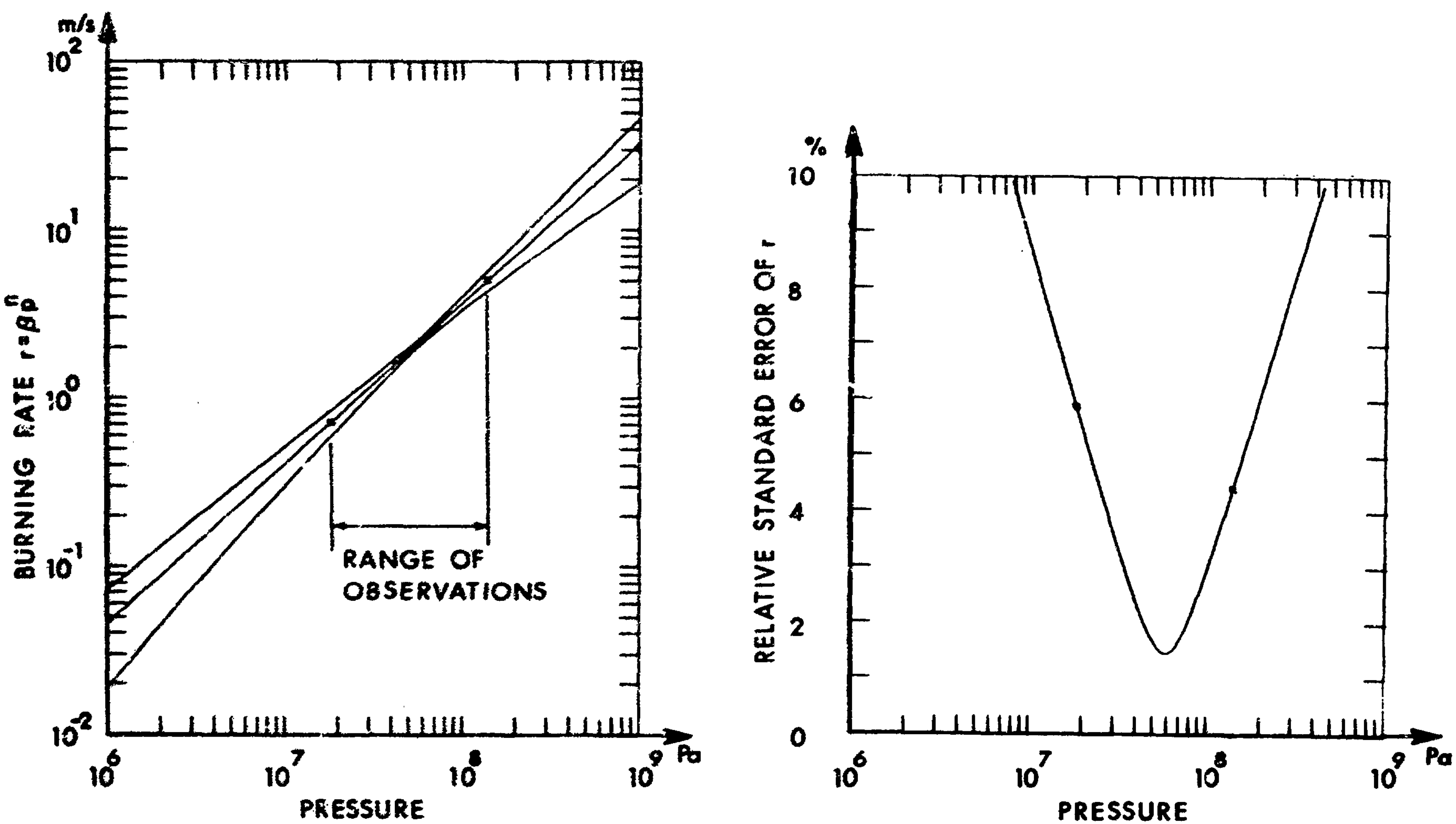

Figure 11c. Burning Rate $r=\beta p^n$ From Experiment 167-3

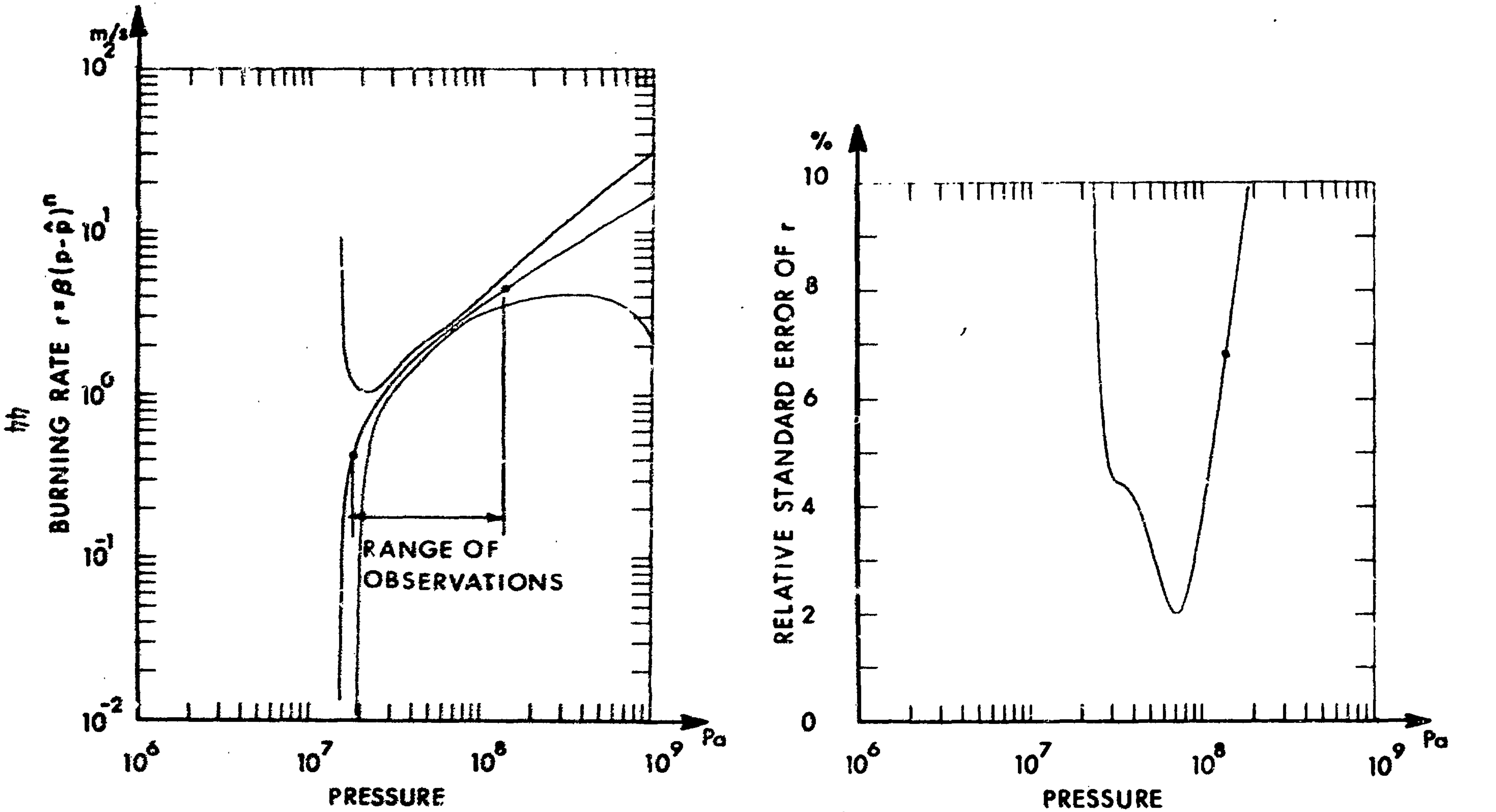

Figure 11d. Burning Rate $r=\beta(p-\hat{p})^n$ From Experiment 167-3

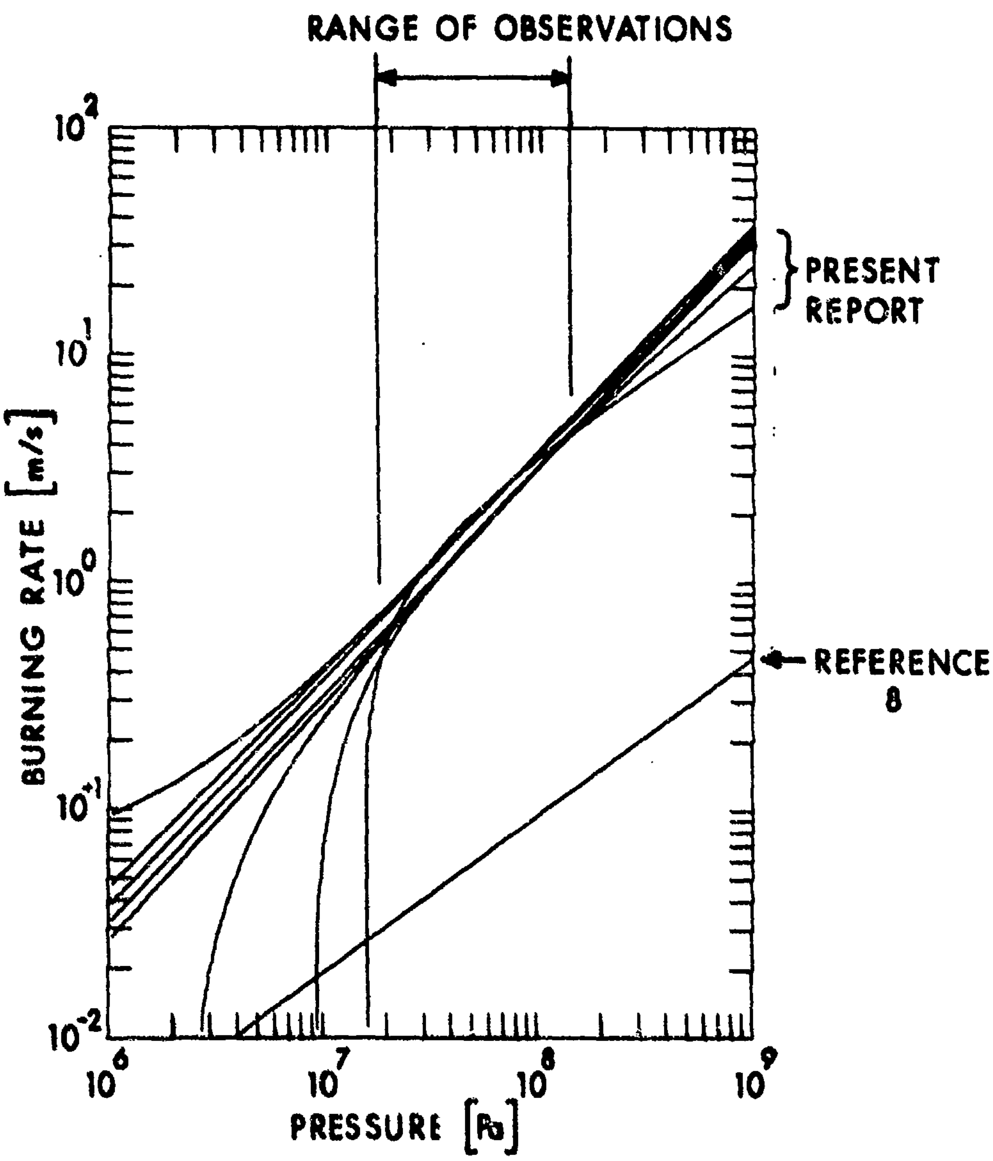

Figure 12 JPN Burning Rate Curves

correlated. The standard error of the burning rate formula was found to increase rapidly outside the observed pressure range. Closed-bomb measurements should be planned therefore so that a large pressure range is covered by the experiment, including particularly those pressures for which the burning rate formula is to be used.

The burning rate of JPN was determined from the experiments 167-3 and 167-4. It was found that for pressures between 20MPa and 200MPa, JPN burning rate can be approximated by the linear function $r = \beta p$ with an accuracy compatible with the accuracy of the measurements. The burning rate outside the above mentioned pressure interval cannot be determined from the given data.

The least squares values of the two parameters in the burning rate formula $r = \beta p^n$ have a correlation coefficient very close to minus one. This strong correlation indicates that one of the parameters is redundant for the data fitting process and might be chosen within limits by other considerations. In the present case $n = 1$ is suggested for simplicity. Equally good agreements with data could be obtained, e.g., by chosing $n = 0.9$ or $n = 1.1$.

Several general improvements of the described closed bomb data reduction process are possible. Thus, instead of evaluating each experiment separately, all experiments involving the same propellant could be evaluated simultaneously. The least squares utility programs presently available at BRL can be used for such purpose. The processing requires only the coding of some new subroutines representing constraint functionals.

Methods for the evaluation of the noisy measurements and for a joint evaluation of data for all pressures can also be developed. In connection with such a joint evaluation more flexible two- and three-parameter burning rate formulas should be tested numerically.

ACKNOWLEDGEMENT

The author is indebted to Mr. John D. Knapton, Interior Ballistics Laboratory, BRL for the permission to use the closed bomb data presented in this report, as well as for suggestions and corrections regarding the theoretical part of the paper.

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